Technical paper

What flow rates can go through a drainage system?

A theoretical background

MSc (Ir.) Marc Buitenhuis MTD

Research Engineer Hydro-Dynamics
Aliaxis
The Netherlands
10/2017

Abstract

Two-phase flow through a pipe has several flow patterns that will take place depending on the circumstances in the system. The flow patterns that are favorable for the integrity of the system, annular for vertical stacks and separated flow for (nearly) horizontal pipes, are limited by the flow rate depending on the pipe diameter. The maximum flow rates that can theoretically be handled by a vertical drainage system are determined. For the horizontal branches the obtained equations appear to be in line with the equations described in NEN3215, for the vertical stack the EN3215 describes a simple experimental formula valid for a conventional system, while the more complex theory for annular flow gives values that exceed the numbers of the NEN formula by far, incorporating one assumption following from experiments [the maximum filling degree of a pipe with water is assumed to be ¼]. Although the maximum possible flow rates inside a pipe can be derived by the formulas there is still a practical limitation that needs to be determined, which is the transition regions from horizontal to vertical flow and vice versa. How this is solved has a great influence on the performance of the system. But with the values determined, at least the maximum achievable limits can be determined, which indicates an upper limit for the system and how close your solution is to the maximum achievable performance.

Context of this paper

This technical paper is part of a library op technical papers. Refer to the below overview of all our technical papers and click on the title for a digital link.

Research | Relevance | Design | Solutions | Materials | Installation | Terminology | Standards

Introduction

As in any other building soil and waste water has to be transported out from a high-rise building towards the urban sewage system. In a high-rise building this means that the wastewater mostly has to travel a long vertical distance before reaching ground level, where it is transported further to the sewage system. The velocities, accelerations and decelerations of the wastewater during this fall leads to additional challenges for the system designer, since these are accompanied by pressure spikes that could put the system integrity at risk: water traps can be sucked empty or blown out resulting in foul odors and health risks.

The challenge for the system designer thus is to balance the system pressures that can occur during operation. One way of doing this is to keep the whole system ventilated, which can be done by creating an open path for air to travel freely to and from all locations in the system. In any horizontal branches of soil and waste systems without special measures (air admittance valves or ventilation stacks) this is achieved by keeping the flow within limits so that the water is running at the bottom of the pipe and air can travel freely at the top of the pipe. For the vertical stack flow it means that a so called annular flow must be maintained. This article will give the reader some insight in the miraculous world of two-phase flow, the combined flow of liquid and gas, to obtain some basic understanding of the way works.

The article will start of by presenting the possible flow patterns that can arise in two-phase flow and what patterns should be avoided to maintain the integrity of the system. In the following part the fluid dynamics equations for the preferred flow patterns for the vertical wastewater flow will be presented and from this the maximum possible flow rates through the various parts of the system will be determined. For the horizontal branches this will lead to the equations described in the EN 12056 standard.

Two-phase flow regimes

A flow of two or more fluids is referred to as multi-phase flow. When only two fluids are present the term two-phase flow is used.

In two-phase flow in pipes several regimes are distinguished governed by the volume fractions, densities and velocities of the two phases. In most situations one of the fluids is a liquid while the other one is a gas, as is the case in the waste water drainage system, where the liquid is water and the gas is air. Beware that the liquid phase is not always pure water, but can contain impurities like soap, sand, etc. In case of toilet flushes a third often solid phase can be present in terms of faeces, toilet paper, etc.

The first focus will be on the two-phase flow of water and air, which is already rather complicated.

We will first focus on vertical pipe flow. When the volume fraction of the gaseous substance is very low we speak of bubbly flow. When the volume fraction is higher and the velocities of the bubbles cause them to coalesce the flow will develop to plug or slug flow, plugs or slugs being large bubbles, large with respect to the dimensions of the system in which the two-phase flow is present and intermitted by. On the other side of the spectrum is the disperse flow, where droplets of liquid are present in the gas. In the midst of these are the separated flows (slug, churn and annular), where both the liquid and the gas are flowing as a continuous phase only influencing each other at the boundary surfaces of the two separate phases.

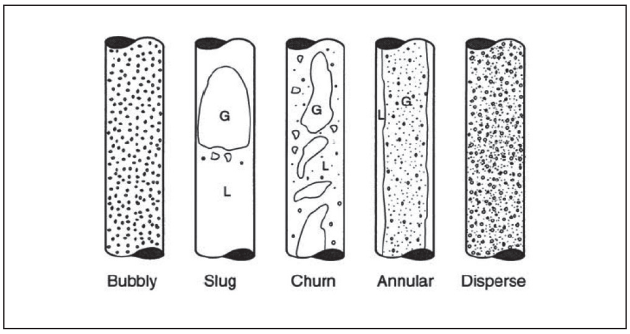


Figure 1.

For a proper working of the waste drainage system the pressure at every location in the system should be around zero, which means the system should be ventilated and thus in contact with ambient air. This means that the air should be a continuous phase through the whole system without interruption. For the vertical flow this means that only disperse or annular flow are permitted and churn, slug or bubbly flow should be prevented. In practice this means that a certain flow rate should not be exceeded. Below and at this maximum flow rate the water will collect along the wall of the pipe and the core will consist of a mist of air and small water droplets. Above this flow rate the waves running at the surface of the water layer at the wall will get so steep that the flow will get unstable and water will close of the pipe diameter at some points in the flow, turning it into plug flow. Because of the local closure of the pipe diameter the ventilation of the whole system can no longer be guaranteed under these circumstances giving rise to pressure spikes that will put the systems integrity at risk.

For horizontal flow there are two additional flow regimes due to gravity: separated and wavy separated flow, where the liquid is flowing at the bottom with the gas above it.

For the horizontal flow we should prevent plug, slug or bubble flow, while stratified, wavy, disperse and annular flow are permitted to have a ventilated system. In practice stratified and wavy stratified flow limit the branch capacity, while annular flow is hard to obtain due to gravity.

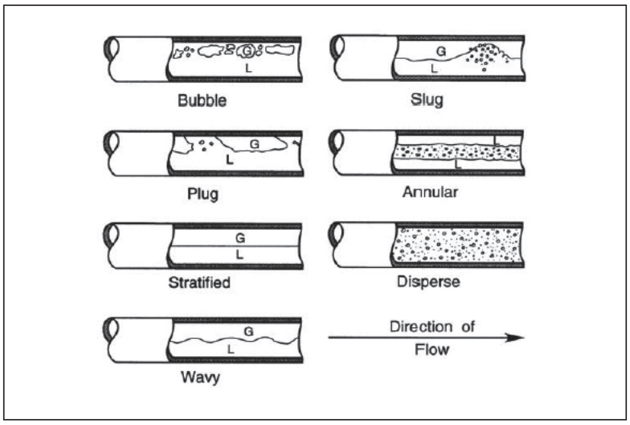
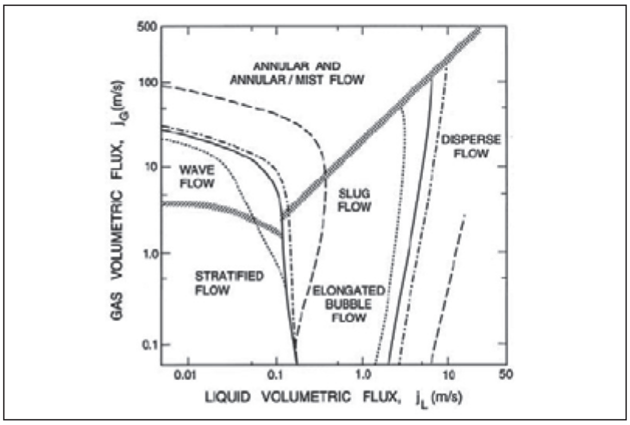
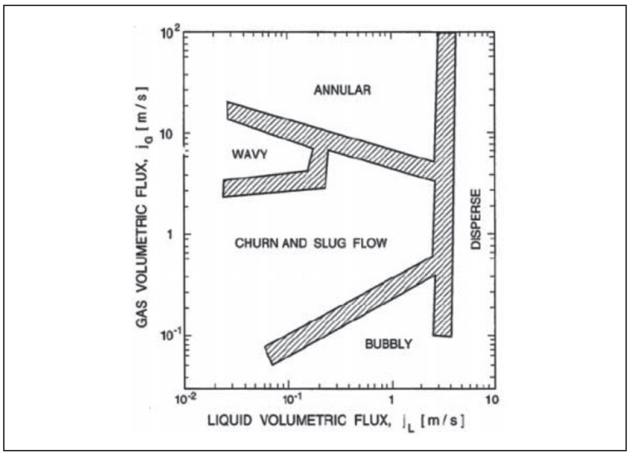


Figure 2.

To avoid unfavorable flow regimes in waste water drainage systems the discharge rate should be limited, the limit depending on the system configuration and pipe diameters involved. For straight vertical pipe of certain diameter graphs of (normalized) gas vs liquid velocity are produced indicating the flow regimes, see fig x below. Be aware that these graphs are quantitatively only valid for the fluids and diameter(s) used in that particular test. Qualitatively however it gives an indication what flow types to expect. It also indicates that there must be a significant gas flow induced by the falling liquid to obtain an annular flow.





Annular Flow model

There is no overall model for two-phase flow. Instead models are developed for each flow regime. For annular flow the model developed is based on the liquid flow down a vertical wall. Below the theory will be presented.

In an annular flow the water reaches a terminal velocity. This is due to the balance in gravity and wall friction forces in the annulus.

Applying Newton's second law leads to the following equation describing the force balance in the annular flow:

$$\rho g\left(\pi D\, t\, dx\right) - \tau_0\, \pi D\, dx = \rho \pi\, D\, t\, dx \frac{dV_w}{dt}$$

With: ρ = density of the liquid
g = gravity coefficient
D = inner pipe diameter
τ_0 = wall shear stress
V_w = flow velocity

When the terminal velocity is reached the right hand term vanishes (since $\frac{dV_w}{dt} = 0$ for the terminal velocity) and thus the equation reduces to:

$$\rho g t = \tau_0 = \frac{1}{2} \rho f V_w^2$$

From this equation both the annulus thickness as the velocity can be deduced:

$$t = \frac{f}{2g} V_w^2$$

$$V_w = \sqrt{\frac{2gt}{f}}$$

t = annular layer thickness
f = friction coefficient

and thus:

$$\frac{1}{\sqrt{f}} = \frac{V_w}{\sqrt{2gt}} = \frac{Q_w}{\pi D t \sqrt{2gt}}$$

Q_w = flow rate

Q_w = flow rate

The Colebrook White equation may be applied with the hydraulic mean depth instead of the pipe diameter resulting in:

$$\frac{1}{\sqrt{f}} = -4 \log\left[\frac{k}{14.84m} + \frac{0.315}{\mathrm{Re}\sqrt{f}}\right]$$

$$where : \mathrm{Re} = \frac{\rho V m}{\mu}$$

meaning that D is replaced by 4 m in the standard equation. The hydraulic mean depth for the annulus is:

$$m = \frac{\pi D t}{\pi D} = t$$

Substituting results in:

$$\frac{V_w}{\sqrt{2gt}} = \frac{Q_w}{\pi D t \sqrt{2gt}} = -4 \log\left[\frac{k}{14.84t} + \frac{0.315 \cdot V_w}{\frac{\rho V_w t}{\mu}\sqrt{2gt}}\right]$$

$$= -4 \log\left[\frac{k}{14.84t} + \frac{0.315 \cdot \mu}{\rho t \sqrt{2gt}}\right]$$

For very smooth walls k=0

The terminal thickness can be determined from the second and last terms. When t has been determined the water velocity can be determined from the first and second term.

The distance required to reach terminal velocity can be deduced by substituting:

$$\frac{dV_w}{dt} = \frac{dV_w}{dz}\frac{dz}{dt} = V_w \frac{dV_w}{dz}$$

in the first equation of this chapter to obtain:

$$\frac{dV_w}{dz} = \frac{1}{V_w}\frac{dV_w}{dt} = \frac{1}{V_w}\left(g - \frac{f}{2}\frac{\pi D}{Q_w} V_w^3\right)$$

From this equation dz can be deduced:

$$dz = \frac{V_w dV_w}{\left(g - \frac{f}{2}\frac{\pi D}{Q_w} V_w^3\right)} = \frac{V_t^2 \frac{V_w}{V_t} d\left(\frac{V_w}{V_t}\right)}{g\left(1 - \frac{V_w^3}{V_t^3}\right)}$$

with:

$$V_t = \frac{Q_w}{\pi D t} \Rightarrow \rho g t = \tau_0 = \frac{1}{2}\rho f V_t^2 = \rho g \frac{Q_w}{\pi D V_t}$$

$$\Rightarrow V_t = \sqrt[3]{\frac{2g}{f}\frac{Q_w}{\pi D}}$$

integration leads to:

$$z = \int \frac{V_t^2 \frac{V_w}{V_t} d\left(\frac{V_w}{V_t}\right)}{g\left(1 - \left(\frac{V_w}{V_t}\right)^3\right)} = 1.56\ \frac{V_t^2}{g}$$

$$= 0.159 \cdot V_t^2$$

Omitting the singularity at $V_w/V_t = 1$ by integrating up to 0.99.

From the above equations the terminal velocity of the falling water and the pipe length to reach this terminal velocity can be obtained. It indicates that the flow will not accelerate endlessly, but reach a terminal velocity and that it will need a limited length of pipe to reach this velocity. Thus it is not so that a longer length of pipe will further accelerate the flow.

From experiments it has been obtained that the annular flow will break up when ¼ of the pipe diameter is filled with water, meaning a water layer t of D/16. Using this experimental value of t will give the maximum velocity and flow rate through the stack at which ventilation is guaranteed.

The table below gives computed flow rate values in l/s for the diameters of single stack systems with stack-aerators:

	Ø110 / D=101,6 mm	Ø160 / D=147,7 mm
k=0	11,0	29,9
k=0,001	5,6	15,5
k=0,001; filling degree = 1/3	14,6	39,8

From the table it can be seen that a roughness factor of 0,001 leads to pessimistic values for a maximum filling degree of ¼ water, since 7.6 l/s is permitted and approved through a stack-aerator Ø110 drainage system. Both taking a higher possible filling degree [1/3] or a lower roughness factor [0] will enhance the maximum flow rate to values that seem more appropriate.

The equation in standard NEN 3215:

$$Q = \gamma_a \cdot s \cdot D^2$$

With: γ_a = 400 m/s
s = 1 for a building height less than 60 m and dependent of height and pipe diameter for higher buildings

does not correspond to the above equations, but to (probably experimental) values for conventional drainage stacks.

Separated flow model

The Colebrook-White equation, Manning equation and Darcy-Weisbach formula for flow through a partially filled inclined pipe or channel can be used to calculate the flow rate in a pipe with a X % filling grade at a slope S:

$$\frac{1}{\sqrt{f}} = 2\log\left[\frac{k}{3.7\cdot D} + \frac{2}{Re}\right.$$

$$V = \frac{1}{n} R_h^{\frac{2}{3}} S^{\frac{1}{2}}$$

$$n = \sqrt{\frac{f}{8g}} R_h^{1/6} = \frac{R_h^{1,}}{C}$$

$$R_h = \frac{A}{P}$$

$$Q = A \cdot V$$

With: f = friction coef

k = roughness c [m]

S = slope of the [m/m]

g = gravity coef

A = wetted area

P = wetted perir

Q = flow rate [m

The flow rate that follows from these calculations can be used to determine the maximum capacity of ventilated and unventilated pipes, assuming a certain filling grade [eg h/d=0.7 for unventilated pipe and h/d=0.95 for ventilated pipe].

The above equations can be rewritten to:

$$Q = AV = \left[X \cdot \frac{\pi}{4} \cdot D^2\right]$$

$$= \left[X \cdot \frac{\pi}{4} \cdot D^2\right.$$

With: X = filling gr pipe in volur

Rh = f(Rh/D)

$$\frac{A}{P\cdot D}\cdot D = \frac{X\cdot\frac{\pi}{4}D^2}{(Y\cdot\pi\cdot D)\cdot D}$$

With: A = the wetted cross sectional area
P = the wetted periphery of the pipe
X = the filling grade in volume percentage
Y = the ratio of the wetted periphery to total pipe periphery
θ = angle corresponding to a filling grade and wetted periphery of pipe diameter

And thus:

$$Q = 10^3 \cdot \left[\left[(\pi - \vartheta/2) + \frac{1}{\pi}\sin\frac{\vartheta}{2}\cdot\cos\frac{\vartheta}{2}\right]\right]$$

$$\cdot\left[\frac{1}{4} + \frac{\left[\sin\frac{\vartheta}{2}\cdot\cos\frac{\vartheta}{2}\right]}{4\cdot(\pi-\vartheta/2)}\right]^{\frac{1}{2}}\cdot\frac{\pi}{4}\cdot\left[C\cdot D^{\frac{5}{2}}\cdot S^{\frac{1}{2}}\right]$$

With Q = flow rate in l/s

With: $C = \gamma_c \cdot \log\frac{3D}{k}$ with $\gamma_c = 18\ m^{0.5}/s$

Using Darcy-Weisbach and Colebrook-White it follows:

$$C = \sqrt{\frac{8g}{f}} =$$

$$\sqrt{8g}\cdot -2\log\left(\frac{k}{3.72D} + \frac{2.51}{Re\cdot\sqrt{f}}\right)$$

$$\cong \sqrt{32g}\log\left(\frac{3.72D}{k}\right) \approx 18\log\left(\frac{3D}{k}\right)$$

For a certain filling grade the part of the equation of the calculation for the flow rate Q before the terms with C, D and S can be totally determined and corresponds to the values given in EN 3215 for unventilated (70% filling grade [h/d = 0.7] resulting in a value for the term of 315) and ventilated pipe (filling grade of 95% [h/d = 0.95] resulting in a value for the term of 395). Besides with k=0.001 generally assumed for pipe roughness, C is a function of D only and thus the flow rate only depends on the inner pipe diameter D and slope S.

Conclusion

From the article presented above it can be concluded that the maximum flow rates that can be handled by an drainage system and its branches can be estimated and are bound to limits arising from fluid dynamics. It can also be concluded that the equations found in the standard NEN 3215 for horizontal branches in the system are directly related to fluid dynamics, while the equation for the stack does apply to a conventional stack configuration and not to an annular flow pattern in the pipe or an drainage system. Yet also for an annular flow pattern there is a maximum flow rate, but the calculated value depends on experimental data for the breakup of the annular flow pattern into a slug pattern. Apart from these limitations there is the transition from horizontal to vertical flow, the inflow from horizontal flow in to the main vertical flow and transition from vertical to horizontal flow at the base of the stack. The way these transitions take place is determining for the limits of the system. Yet the equations deducted in this article are giving upper limits to what flow rates can be handled by a drainage system.

Steve White
Technical Director DWV
Aliaxis High-Rise Building Solutions

MSc (Ir.) Marc Buitenhuis MTD
Research Engineer Hydro-Dynamics
Aliaxis

1. Fox, Robert W., McDonald, Alan T. *Introduction to fluid mechanics*, third edition, 1985, School of Mechanical Engineering Purdue University, John Wiley & Sons

2. John A. Swaffield, Larry S. Galowin, *The engineered design of building drainage systems*, 1992, Ashgate Publishing Limited, Hants (UK)

3. NEN 3215 - 2011

Read more technical papers related to this subject

- Research - What happens at the base of the stack

19/04/029